ISSN 2348-1218 (print)
International Journal of Interdisciplinary Research and Innovations ISSN 2348-1226 (online)
Vol. 7, Issue 2, pp: (587-600), Month: April - June 2019, Available at: www.researchpublish.com

EFFECT OF PRESERVATION TECHNOLOGY ON OPTIMISATION OF TWO WAREHOUSE INVENTORY MODEL FOR DETERIORATING ITEMS WITH STOCK DEPENDENT DEMAND UNDER INFLATION

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Abstract: **This research paper examines the development of two warehouse model under the influence of inflation and use of preservation technology for deteriorating goods. This model depends on the stock dependent demand. In this model, goods stored in a rented warehouse charged higher units holding cost as compare to the owned warehouse. Due to the higher holding cost on rented warehouse's goods, seller invests on preservation technology. Firstly, the inventor in the rented warehouse reaches to the level zero due to demand and deterioration and then the inventory in the owned warehouse reaches to level zero. When the inventory level reaches to zero then the model permits the fulfillment of the goods by using the backlogging depends on the time for the next cycle. This study also determines the retailer's optimal replenishment policy that minimizes total cost per unit time. Total cost function is also affected by the JIT setup Cost. In addition, the research done in this study demonstrates that an optimal solution exists and is unique. A numerical example is displayed for the development of the model and sensitivity analysis has been performed to identify the behavior of model. The results of this study provide managerial insights for enterprises that use a rented warehouse to minimize costs by coordinating lot sizes.**

Keywords: **Inventory, Two-Warehouse, JIT setup cost, Preservation Technology, Partial Backlogging, Deterioration, Shortage and Stock-dependent demand.**

1. INTRODUCTION

In the last 30 years, Many researchers have a focus on the model for inventory policies associated with the two warehouse system. This type of approach in inventory model has been proposed for the first time by Hartely(1976)[9]. In this system it is believed that the cost of keeping good in RW is higher than OW. Hence, goods in RW are first transferred to OW to meet the demand until the stock level in RW drops to zero and then items in OW are released. Since enterprises purchase higher amount of goods than the volume available to carry goods in their own warehouse(OW). So, the enterprises stored excess quantity of goods in an additional storage space, the rented warehouse(RW). The rented warehouse charged higher units holding cost then owned warehouse. Many items deteriorate during the shortage period. deterioration is defined as ``damage, spoilage, decay, obsolescence, evaporation, pilferage, etc., that result in decreasing the usefulness of the original one". Deteriorating rate can be controlled by using preservation techniques during the deteriorating period. Sarma (1983)[23] developed a two-warehouse model for deteriorating items with the infinite replenishment rate and shortages. Sarma (1987) [2] extended Hartely's model to cover the transportation cost from RW to OW that is considered to be a fixed constant independent of the quantity being transported. Pakkala and Achary(1991)[4] further developed a two-

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warehouse model with order level probabilistics inventory for deteriorating items. In this model, they assumed infinite production rate with shortage and also the different deterioration rate for both the warehouses. In all these models, the demand was assumed to be constant and the cost of transporting items from RW to OW was not taken into account. In a recent paper, through employing continuous transportation pattern, Bhunia and Maiti (1998)[2] developed a two warehouse model for deteriorating items with linearly increasing demand and shortages during the infinite period. In another paper, Zhou (1998)[48] presented a two-warehouse model for deteriorating items with time varying demand and shortages during the finite-planning horizon. In this direction many notable papers are published by , Hsieh et al.[7] Singh et al.[8], Jaggi et al.[9].

In most inventory models, demand is persistent, is changing over time or only dependent on price. However, this assumption does not always apply to real situations. For example in the supermarket, it is observed that the items displayed in the store in large quantities attract more consumers and generate high demand. In recent decades, many physicians and researchers have noticed this phenomenon that the demand depends on the inventory displayed in the store. Levin et al. (1963)[17] observed that large piles of consumer items displayed in a supermarket would buy more for consumers. Gupta and Vrat (1986)[8] were first to establish model based on the derivation of EOQ with stock- dependent consumption rate. Inventory models were introduced BY Padmanabhan and Vrat (1995)[20] for the items worn with the stock dependent demand, where demand was considered as the current inventory level work and the rate of decline was continuously taken. A comprehensive review of literature was provided with list-level-dependent demand by Urban (2005)[42].Wu et al. (2006)[44] considered a problem of determining the optimal replenishment policy for non-instantaneous deteriorating items with stock-dependent demand. Changing the objective to obtain the maximal profit, Chang et al. (2010) [7]amended the model of Wu et al. (2006)[44] by setting a maximum inventory level to match realistic circumstance where most retailer outlets had limited shelf space, and relaxing the restriction of zero ending inventory when shortages were not desirable. Soni (2013)[36] formulated an inventory model to seek the optimal sales price and replenishment cycle, where the demand rate was a multivariate function of price and level of inventory, and delay in payment was permissible. Yang (2014) [47]developed an inventory model under a stock-dependent demand rate and stock- dependent holding cost rate with relaxed terminal conditions to determine the optimal order quantity and the ending inventory level. An optimal control model for perishable item was established by Lu et al. (2014)[18] to seek the optimal joint dynamic pricing and replenishment strategy, where the demand rate dependent on the on-hand stocks and the sales price. The reader can study the recent works of Das et al. [34], Niu et al. [35], Rong et al. [36], Dey et al. [37], Maiti [38], Lee aet al. [39], Bhunia et al. [40], Liang et al. [41], Bhunia et al. [1], Yang et al. [47], Jaggi et al. [44], Bhunia et al. [3,4], Jaggi et al. [14] and Tiwari et al. [41]. Table 1 presents a comparison of some recent papers related to inventory models considering one- or two-warehouse system.

Table 1: Comparison of different inventory models based on warehouse.

References	Preservation Technology	Demand	Backlogged	Inflation	Holding Cost	Warehouse
Liang et al.	No	Constant	Partial	No	Constant	Two
Bhumia et al.	No	Linear	Partial	No	Constant	Two
Yang et al.	No	Constant	Full	No	Constant	Two
Jaggi et al.	No	Selling Price	partial	No	Constant	Two
Bhumia et al.	No	Advertise Demand	Partial	No	Constant	Two
Bhumia et al.	No	Selling Price	Partial	No	Constant	Two
Jaggi et al.	No	Constant	No	No	Constant	Two
Tiwari et al.	No	Constant	Full	Yes	Constant	Two
Taleziadeh et al.	No	Constant	No	No	Fuzzy, Crisp	Single
Taleziadeh et al.	No	Constant	No	No	Constant	Single
Liao et al.	No	Constant	No	No	Constant	Two
Chakrabarty et al.	No	Exponentially Incresing time	Partial	Yes	Constant	Two
This Paper	Yes	Stock Dependent	Partial	Yes	Variable	Two

The concept of inflation has been first studied by Buzacott [5]. After that, many authors have extended the work of Buzacotta. Wee et al. [43] and Yang [46] focused on two-warehouse problem with partially backlogged shortages under the effect of inflation. Singh and Rathore [33-35] have studied the effect of preservation technology in an inflationary environment. Many other authors have considered the effect of inflation in their inventory control modeling, like Singh et al. [36], Singh et al. [37], Patra and Ratha [22], Sarkar et al. [39-40], Sarkar et al. [41], Singh and Rathore [34-35], etc. Recently, Liao et al. (2012) generalized Goyal's EOQ model to allow for deteriorating items with two warehouses (i.e., an owned warehouse (OW) with the maximum storage capacity of W, and a rented warehouse (RW) with unlimited storage capacity) under an order size dependent trade credit. In this paper, we attempt to overcome some shortcomings in their model such as (1) the cost of deteriorating items was included twice in their objective function, (2) we substitute the unit purchase cost by unit selling price in calculations of interest earned, and (3) they erroneously considered the time taken by inventory in RW to reduce to zero, to be same as the time taken by total inventory to reduce to the storage capacity of the OW.

The numerical examples are resolved to display the applicability of the proposed inventory model, after which the sensitivity analysis of the optimal solution in relation to the input parameters of the inventory system. The proposed inventory model is useful because it helps decision makers in making significant recurrence decisions. The structure of this paper is as follows. Section 2 gives detailed information and notation used in paperwork. Section 3 presents the creation of inventory models. Section 4 resolves some numerical examples to validate the inventory model, and the effectiveness of the proposed inventory model is further illustrated through a comprehensive sensitivity analysis. Finally, Section 5 provides some conclusions and future research instructions.

2. ASSUMPTION AND NOTATION

Based on the data of Chaudhari et al.[15] model, this paper is developed with the following notations and assumptions.

NOTATIONS

Co	JIT setup cost per replenishment
D	Demand rate per unit time
θ	Constant deterioration rate
W	Storage capacity of OW
T	Length of each replenishment cycle
S	Selling cost per unit
C	Purchasing cost per unit
$I_R(t)$	Level of inventory at t time in RW
$I_o(t)$	Level of inventory at t time in RW
h_o	Holding cost per unit time Charged by OW
h_r	Holding cost per unit time Charged by RW
t_w	Time at which the inventory level reaches zero in RW
M	Credit period set by the supplier
I_o	Capital opportunity cost (as a percentage)
I_e	Earned interest rate (as a percentage)
H	Length of planning horizon
r	Discount rate, representing the time value of money
R	r-i, Constant discount rate of inflation
t_j	Time at which inventory level in jth replenishment cycle drops to zero
T_j	Total time that is elapsed up to and including the jth replenishment cycle, $T_o = 0$
m(ξ)	The preservation technology $\tau_p > 0$
τ_p	Resultant deterioration rate $= (\theta - m(\xi))$

C_s	Shortage cost per unit time
F	Fraction of replenishment cycle where net stock is positive (decision variable)
N	Number of replenishment during the planning horizon, N = H/T
Q	Item's order quantity

ASSUMPTIONS

1. The mathematical model in this study is based on the following assumptions:
2. The planning horizon in this model is finite.
3. The deteriorating rate of the item is decreasing as the preservation effort increases.
4. The demand rate is stock - dependent and taken as D(t) = (a + b I(t)), a, b > 0.
5. To stored goods OW has fixed space while RW has unlimited space.
6. The items in OW are consumed only after consuming the items kept in RW.
7. Shortages are allowed and partially backlogged where backlogging rate is B(t) = $\frac{1}{1+\delta(T-t)}$, where δ is a positive constant and (T-t) is the waiting time for next replenishment to start at time T.
8. In period [t_1, T] the demand D(a+ bI(t)) at any time t is satisfied whereas the remaining part of the demand D(1-$\frac{1}{1+\delta(T-t)}$) remains unsatisfied, where δ is a positive constant and (T- t) is the waiting time for next replenishment to start at time T. So, the backlogging rate is $\frac{D}{1+\delta(T-t)}$ which varies as function of waiting time.
9. At the end of the planning horizon, inventory level will be zero.
10. The number of replenishment are restricted to integer one.
11. The inventory costs in RW are higher than those in OW.
12. The last order is only being placed to satisfy the shortage of last period.
13. The proportion of reduced deterioration rate, τ_p, is a continuous, increasing function of retailer's capital investment.

3. MATHEMATICAL MODELLING

Development of Two warehouse Model **($t_a < t_1$)**

The system involves two warehouse model with Q units of items. At first W units are stored in OW and the rests are kept in RW. So, during the time interval [0, t_w], the items in RW decreases due to exponential demand rate and reached to the level zero. Deterioration is being disposed at a rate of (θ - m(ξ))for a given preservation investment ξ. During [0, t_w], the inventory level on OW is depleted to deterioration. Then during [t_w, t_1] the item in OW reached to the level zero due to demand and deterioration. During the interval [t_1, T] shortages of the items start, demand is partially backlogged with backlogging rate $\frac{1}{1+\delta(T-t)}$ and after fulfilling the demand new replenishment cycle starts. The inventory situation is represented in Figure 1.

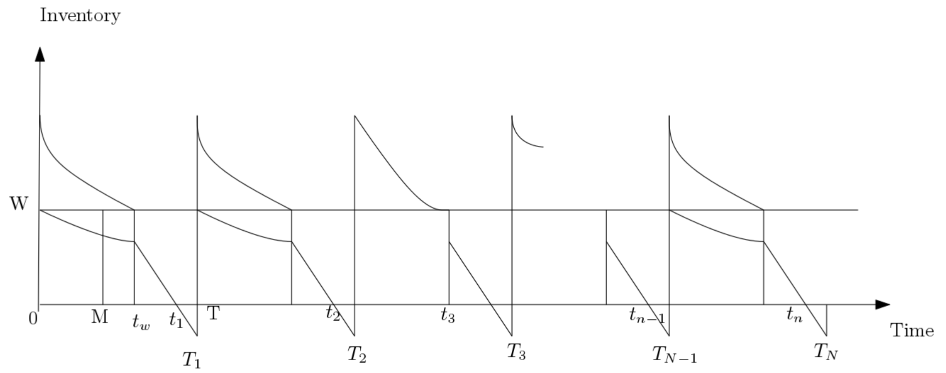

Figure 1: Graphical representation of inventory cycle

Therefore, the differential equations that describe the inventory level in the RW and OW at time t over the period [0, T] are given by:

$$\frac{dI_R(t)}{dt} + \tau_p I_R(t) = -(a+bI_R(t)); \quad 0 \le t \le t_w \tag{1.1}$$

$$\frac{dI_O(t)}{dt} = -\alpha I_O(t); \quad 0 \le t \le t_w \tag{1.2}$$

$$\frac{dI_O(t)}{dt} + \alpha I_O(t) = -(a+bI_R(t)); \; t_w \le t \le t_1 \tag{1.3}$$

$$\frac{dI_O(t)}{dt} = -\frac{a}{1+\delta(T-t)}; \; t_1 \le t \le T \tag{1.4}$$

With initial and boundary condition $I_R(t_W) = 0 = I(t_1)$, $I_O(0) = W$ (1.5)

The solution of above equation (1.1)-(1.4) subject to the conditions (1.5) are given by

$$I_R(t) = \frac{a}{b+\tau_p}\left(e^{(b+\tau_p)(t_W-t)} - 1\right); \; 0 \le t \le t_w \tag{1.6}$$

$$I_O(t) = We^{-\alpha t}; \; 0 \le t \le t_w \tag{1.7}$$

$$I_O(t) = \frac{a}{b+\alpha}\left(e^{(b+\alpha)(t_1-t)} - 1\right); \; t_W \le t \le t_1 \tag{1.8}$$

$$I(t) = \frac{a}{\delta}\log\frac{1+\delta(T-t)}{1+\delta(T-t_1)}; \; t_1 \le t \le T \tag{1.9}$$

The maximum inventory at the beginning of each cycle:

$$I_m = I_R(0) + I_O(0) = \frac{a}{b+\tau_p}\left(e^{(b+\tau_p)tw} - 1\right) + W \tag{1.10}$$

The maximum shortage at the beginning of each cycle:

$$I_s = -I(T) = \frac{a}{\delta}\, log(1+\delta(T-t_1))$$
(1.11)

3.1. JIT Setup Cost

Since there is N number of replenishment, So the JIT setup cost over the planning horizon with inflation is

$$C_{SET} = \sum_{j=1}^{N} C_O \frac{(a-1)}{bI_m} e^{-JRT} = C_O \frac{(a-1)}{bI_m}\left(\frac{e^{-RH}-1}{e^{\frac{-RH}{N}}-1}\right)$$
(1.12)

3.2. Holding Cost

Holding cost in RW

$$C_{RW} = \int_0^{t_w} (h_r + \lambda t) I_R(t)dt$$

$$= \frac{ah_r}{a+\tau_p}\left(\frac{e^{(b+\tau_p)t_W}}{\tau_p+b} - \frac{1}{\tau_p+b} - t_w\right) + \frac{a\lambda}{b+\tau_p}\left(-\frac{t_w}{\tau_p+b} - \frac{1}{(\tau_p+b)^2} - \frac{t_W^2}{2} + \frac{e^{(\tau_p+b)t_W)}}{(\tau_p+b)^2}\right)$$

Holding cost in OW

$$C_{OW} = h_o\left(\int_0^{t_w} I_O(t)dt + \int_{t_w}^{t_1} I_O(t)dt\right)$$

$$= \frac{h_0 W}{\alpha}(1-e^{\alpha t_w}) - \frac{ah_o}{b+\alpha}\left(\frac{1}{\alpha+b}(1-e^{(b+\alpha)(t_1-t_w)} + (t_1-t_w)\right)$$

Total holding cost

$$C_{HC} = C_{RW} + C_{OW}$$

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$= \frac{ah_r}{a+\tau_p}\left(\frac{e^{(b+\tau_p)t_w}}{\tau_p+b} - \frac{1}{\tau_p+b} - t_w\right) + \frac{a\lambda}{b+\tau_p}\left(-\frac{t_w}{\tau_p+b} - \frac{1}{(\tau_p+b)^2} - \frac{t_w^2}{2} + \frac{e^{(\tau_p+b)t_w}}{(\tau_p+b)^2}\right)$

$+\frac{h_0 W}{\alpha}(1-e^{\alpha t_w}) - \frac{ah_o}{b+\alpha}\left(\frac{1}{\alpha+b}(1-e^{(b+\alpha)(t_1-t_w)} + (t_1-t_w)\right)$

(1.13)

Total holding cost over planning horizon with inflation is

$C_{HCI} = \sum_{J=1}^{N-1}(C_{RW} + C_{OW})e^{-JRT}$

$= \left(\frac{ah_r}{a+\tau_p}\left(\frac{e^{(b+\tau_p)t_w}}{\tau_p+b} - \frac{1}{\tau_p+b} - t_w\right) + \frac{a\lambda}{b+\tau_p}\left(-\frac{t_w}{\tau_p+b} - \frac{1}{(\tau_p+b)^2} - \frac{t_w^2}{2} + \frac{e^{(\tau_p+b)t_w}}{(\tau_p+b)^2}\right) + \frac{h_0 W}{\alpha}(1-e^{\alpha t_w}) -\right.$

$\left.\frac{ah_o}{b+\alpha}\left(\frac{1}{\alpha+b}(1-e^{(b+\alpha)(t_1-t_w)} + (t_1-t_w)\right)\right)\left(\frac{e^{-RH}-1}{e^{\frac{-RH}{N}}-1}\right)$

(1.14)

Putting T = $\frac{H}{N}$ and $t_1 = \frac{FH}{N}$

$C_{HCI} = \left(\frac{ah_r}{a+\tau_p}\left(\frac{e^{(b+\tau_p)t_w}}{\tau_p+b} - \frac{1}{\tau_p+b} - t_w\right) + \frac{a\lambda}{b+\tau_p}\left(-\frac{t_w}{\tau_p+b} - \frac{1}{(\tau_p+b)^2} - \frac{t_w^2}{2} + \frac{e^{(\tau_p+b)t_w}}{(\tau_p+b)^2}\right) + \frac{h_0 W}{\alpha}(1-e^{\alpha t_w}) -\right.$

$\left.\frac{ah_o}{b+\alpha}\left(\frac{1}{\alpha+b}(1-e^{(b+\alpha)\left(\frac{FH}{N}-t_w\right)} + \left(\frac{FH}{N}-t_w\right)\right)\right)\left(\frac{e^{-RH}-1}{e^{\frac{-RH}{N}}-1}\right)$

(1.15)

3.3. Shortage Cost

Average shortage S should be determined at first

$S = \frac{1}{2}\int_{t_1}^{T}\frac{a}{\delta}\,log(1+\delta(T-t_1))dt$

$= \frac{1}{2}\frac{a}{\delta}\,log(1+\delta(T-t_1))(T-t_1)$ (1.16)

Total storage cost over planning horizon with inflation

$C_{SCI} = \sum_{J=1}^{N-1} SC_S e^{-JRT}$

$= \frac{1}{2}\frac{a}{\delta}\,log(1+\delta(T-t_1))(T-t_1)\left(\frac{e^{-RH}-1}{e^{\frac{-RH}{N}}-1}\right)$

Putting T = $\frac{H}{N}$ and $t_1 = \frac{FH}{N}$

=

$\frac{1}{2}\frac{a}{\delta}\,log\left(1+\delta\left(\frac{H}{N}-\frac{FH}{N}\right)\right)\left(\frac{H}{N}-\frac{FH}{N}\right)\left(\frac{e^{-RH}-1}{e^{\frac{-RH}{N}}-1}\right)$ (1.17)

3.4. Purchasing Cost

Purchasing cost of the j[th] cycle C_{pj} is calculated as

$$C_{pj} = CI_m + CI_s e^{-T}$$

$= \frac{Ca}{b+\tau_p}(e^{(b+\tau_p)tw}-1) + CW + C\left(-\frac{a}{\delta}\log\frac{1+\delta T}{1+\delta t_1}\right)e^{-T}$ (1.18)

Total purchasing cost over planning horizon with inflation is

$C_{PUR} = \sum_{J=1}^{N-1} C_{pj}e^{-JRT}$

$= \left(\frac{Ca}{b+\tau_p}(e^{(b+\tau_p)tw}-1) + CW + C\left(-\frac{a}{\delta}\log\frac{1+\delta T}{1+\delta t_1}\right)e^{-T}\right)\left(\frac{e^{-RH}-1}{e^{\frac{-RH}{N}}-1}\right)$

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Putting T = $\frac{H}{N}$ and $t_1 = \frac{FH}{N}$

$$= (\frac{Ca}{b+\tau_p}(e^{(b+\tau_p)tw} - 1) + \text{CW} + C(-\frac{a}{\delta}\log\frac{1+\delta\frac{H}{N}}{1+\delta\frac{FH}{N}})e^{-\frac{H}{N}})(\frac{e^{-RH}-1}{e^{\frac{-RH}{N}}-1}) \quad (1.19)$$

3.5. Interest earned and charged $(M < t_w < t_1)$

Since, the retailer sells the goods and continuously accumulates the sales revenue and earns interest with rate I_e during the period 0 to M. Again, the retailer starts paying interest for the items in stock after M time with I_p rate.

$$\text{I.E} = I_e S\left(I_s M + \int_0^M Dtdt\right)$$

$$= I_e S\,[(\frac{a}{\delta}\log\frac{1+\delta T}{1+\delta t_1}))M + \frac{aM^2}{2}] \quad (1.20)$$

Total Interest earned over the planning horizon with inflation is

$$\text{TIE} = \sum_{J=1}^{N-1} I.E\, e^{-JRT}$$

$$= I_e S\,[(\frac{a}{\delta}\log\frac{1+\delta T}{1+\delta\frac{FH}{N}}))M + \frac{aM^2}{2}]\,(\frac{e^{-RH}-1}{e^{\frac{-RH}{N}}-1})$$

Putting T = $\frac{H}{N}$ and $t_1 = \frac{FH}{N}$

$$= I_e S\,[(\frac{a}{\delta}\log\frac{1+\delta\frac{H}{N}}{1+\delta\frac{FH}{N}}))M + \frac{aM^2}{2}]\,(\frac{e^{-RH}-1}{e^{\frac{-RH}{N}}-1})$$

(1.21)

Interest payable is

$$\text{I.C} = I_c C\int_M^{t_1} I(t)dt = I_c C(\int_M^{tw} I_R(t)dt + \int_M^{tw} I_o(t)dt + \int_{t_w}^{t_1} I_o(t)dt)$$

Total interest payable over planning horizon with inflation

$$\text{TIC} = \sum_{J=1}^{N-1} IC\, e^{-JRT}$$

$$= \text{IC}(\frac{a}{b+\tau_p}(\frac{1}{\tau_p+b}(e^{(b+\tau_p)(t_w-M)} - 1) - (t_w - M)) - \frac{1}{b}(e^{bt_w} - e^{bM}) - \frac{W_1}{\alpha}(e^{-\alpha t_w} - e^{-\alpha M}) + \frac{a}{b+\alpha}(-\frac{1}{\alpha+b}(e^{(b+\alpha)(t_1-t_w)} - (t_1 - t_w)))(\frac{e^{-RH}-1}{e^{\frac{-RH}{N}}-1})$$

(1.22)

Putting T = $\frac{H}{N}$ and $t_1 = \frac{FH}{N}$

$$= \text{IC}(\frac{a}{b+\tau_p}(\frac{1}{\tau_p+b}(e^{(b+\tau_p)(t_w-M)} - 1) - (t_w - M)) - \frac{1}{b}(e^{bt_w} - e^{bM}) - \frac{W_1}{\alpha}(e^{-\alpha t_w} - e^{-\alpha M}) + \frac{a}{b+\alpha}(-\frac{1}{\alpha+b}(e^{(b+\alpha)(\frac{FH}{N}-t_w)} - (\frac{FH}{N} - t_w)))(\frac{e^{-RH}-1}{e^{\frac{-RH}{N}}-1})$$

(1.22)

Now, Total cost function

TC(F) = $C_{SET}+C_{HCI}+C_{SHI}+C_{PUR}$+TIC-TIE

$$= C_O\frac{(a-1)}{bI_m}(\frac{e^{-RH}-1}{e^{\frac{-RH}{N}}-1}) + (\frac{ah_r}{a+\tau_p}\left(\frac{e^{(b+\tau_p)t_w}}{\tau_p+b} - \frac{1}{\tau_p+b} - t_w\right) + \frac{a\lambda}{b+\tau_p}(-\frac{t_w}{\tau_p+b} - \frac{1}{(\tau_p+b)^2} - \frac{t_w^2}{2} + \frac{e^{(\tau_p+b)t_w)}}{(\tau_p+b)^2}) + \frac{h_0 W}{\alpha}(1-e^{\alpha t_w}) -$$

$$\frac{ah_o}{b+\alpha}\left(\frac{1}{\alpha+b}\left(1-e^{(b+\alpha)\left(\frac{FH}{N}-t_w\right)}+\left(\frac{FH}{N}-t_w\right)\right)\right)\left(\frac{e^{-RH}-1}{e^{\frac{-RH}{N}}-1}\right)+\frac{1}{2}\frac{a}{\delta}\log\left(1+\delta\left(\frac{H}{N}-\frac{FH}{N}\right)\right)\left(\frac{H}{N}-\frac{FH}{N}\right)\left(\frac{e^{-RH}-1}{e^{\frac{-RH}{N}}-1}\right)+\left(\frac{Ca}{b+\tau_p}\left(e^{(b+\tau_p)tW}-1\right)\right.$$
$$+\quad CW+C\left(-\frac{a}{\delta}\log\frac{1+\delta\frac{H}{N}}{1+\delta\frac{FH}{N}}\right)e^{-\frac{H}{N}}\quad)\left(\frac{e^{-RH}-1}{e^{\frac{-RH}{N}}-1}\right)\quad+\quad IC\left(\frac{a}{b+\tau_p}\left(\frac{1}{\tau_p+b}\left(e^{(b+\tau_p)(t_w-M)}-1\right)-(t_w-M)\right)\text{-}\frac{1}{b}\left(e^{bt_w}-e^{bM}\right)-\right.$$
$$\frac{W_1}{\alpha}\left(e^{-\alpha t_w}-e^{-\alpha M}\right)+\quad\frac{a}{b+\alpha}\quad\left(-\frac{1}{\alpha+b}\left(e^{(b+\alpha)\left(\frac{FH}{N}-t_w\right)}-\left(\frac{FH}{N}-t_w\right)\right)\right)\left(\frac{e^{-RH}-1}{e^{\frac{-RH}{N}}-1}\right)\quad+\quad I_eS\quad\left[\left(\frac{a}{\delta}\log\frac{1+\delta\frac{H}{N}}{1+\delta\frac{FH}{N}}\right)\right)M+\frac{aM^2}{2}\right]\left(\frac{e^{-RH}-1}{e^{\frac{-RH}{N}}-1}\right)$$

(1.23)

Since the total cost function is very small and complex to differentiate by hand, so we solved it by using Mathematica Software. So, the sufficient condition $\frac{\partial^2 TC(F)}{\partial F^2} > 0$ must satisfied.

4. NUMERICAL EXAMPLE

To illustrate the proposed model, we consider some examples as:

***Example* 1:** Let us take the parameter values in the inventory model system as h_o = \$0.02, h_r = \$0.05, M =1/12, W = 30, α =.02, C_o = \$15, C_s = \$1.3, τ_p = .02, R = 0.50, δ= 0.9, H = 2, I_e = \$.35, I_c = \$ 0.5, N = 2, a = 300, b = 4, s = 7, t_w=.05 and $\lambda =$ .05 in suitable units. The optimal solution is TC(F) = 2849.97, F* = 0.331844(Fig. 2).

***Example* 2:** Let us take the parameter values in the inventory model system as h_o = \$0.2, h_r = \$0.5, M =1/15, W = 30, α =.01, C_o = \$50, C_s = \$0.3, R = 0.30, δ= 0.9, H = 2, I_e = \$.08, I_c = \$ 0.1, N = 9, a = 200, b = 2, s = 7, t_w=.05 and $\lambda =$ .05 in suitable units. The optimal solution is TC(F) = 9602.85, F* = 1.18231(Fig. 3).

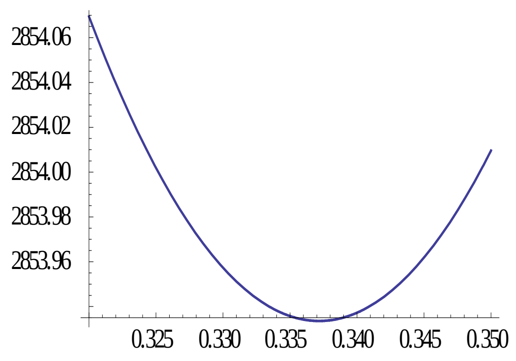


Figure 2: Total cost function TC versus F of Example 1

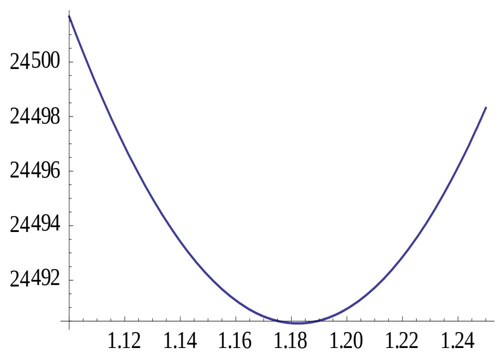


Figure 3: Total cost function TC versus F of Example 2

Table 1: Some data based on numerical example

Parameter	Values	Different values of N	F*	TC(F*)
α	.05	7	.331	2850.01
	.05	6	.340	1927.38
τ_p	.07	7	.332	2849.51
	.07	6	.341	1926.94
R	0.4	7	.332	3476.74
	0.4	6	.341	2333.66
M	1/14	7	.337	3481.10
	1/14	6	.337	2337.77

Table 2: Some data based on numerical example

Parameter	Values	Different values of N	F*	TC(F*)
α	.04	10	1.25644	12258.9
	.04	8	1.09581	7439.84
τ_p	.05	10	1.26054	12239.7
	.05	8	1.09959	7423.6
R	0.1	10	1.26054	24899.2
	0.1	8	1.09959	14378.3
M	1/18	10	1.26125	12255.1
	1/18	8	1.09959	7435.78

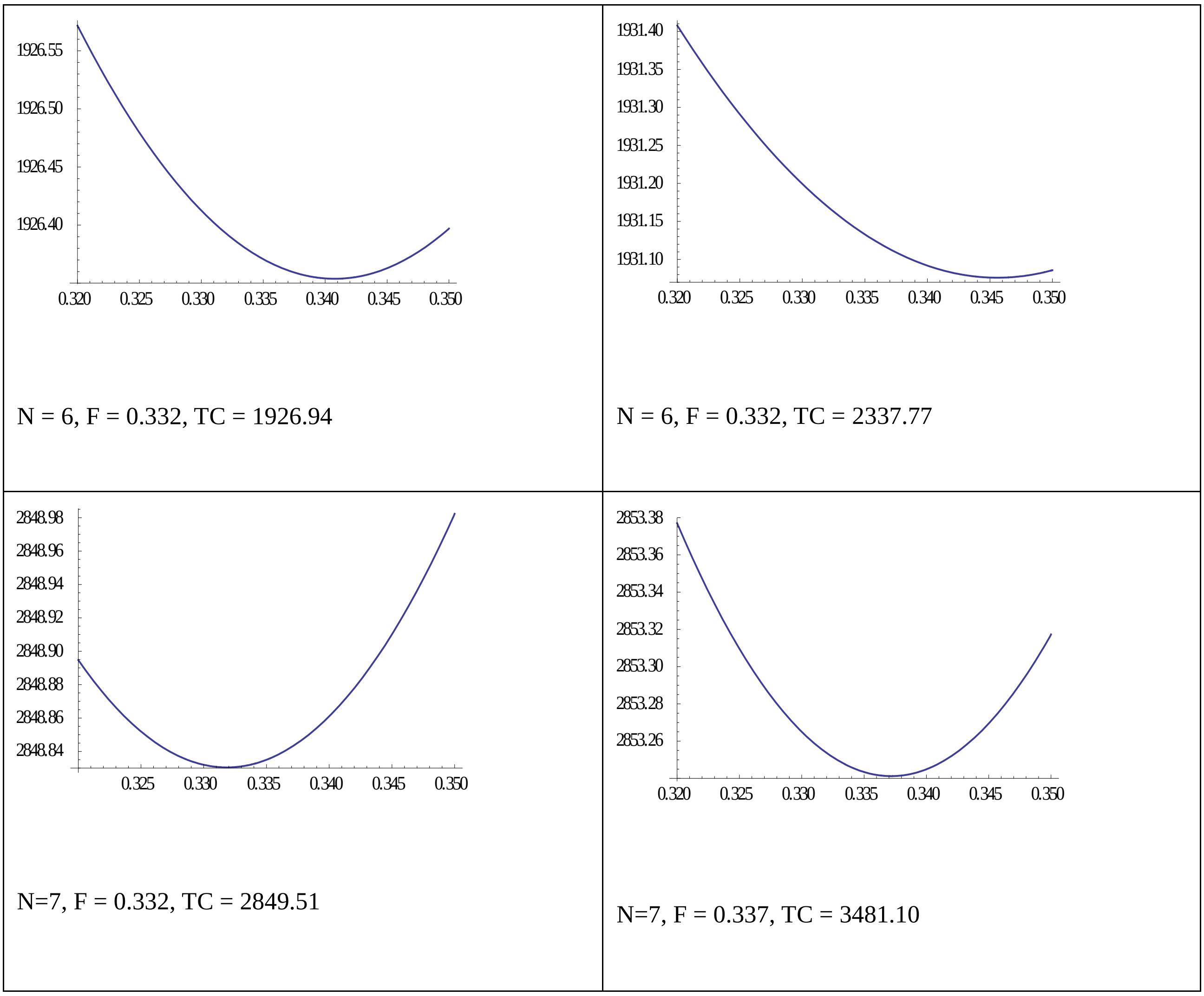
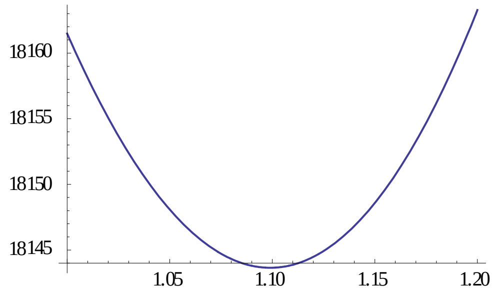
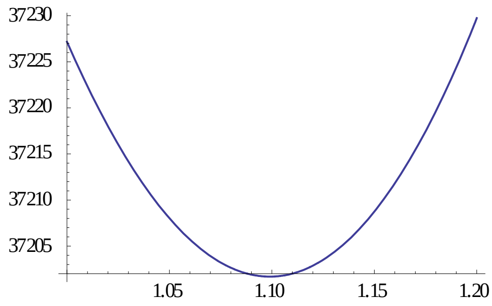


Figure 4: Showing some best results of example 1

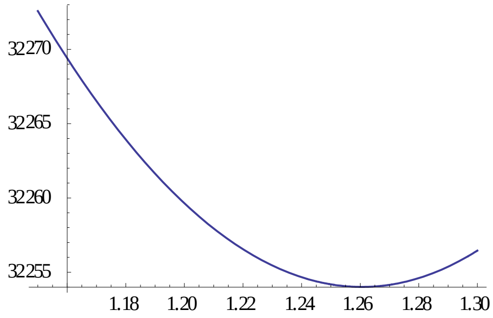
ISSN 2348-1218 (print)
International Journal of Interdisciplinary Research and Innovations ISSN 2348-1226 (online)
Vol. 7, Issue 2, pp: (587-600), Month: April - June 2019, Available at: www.researchpublish.com



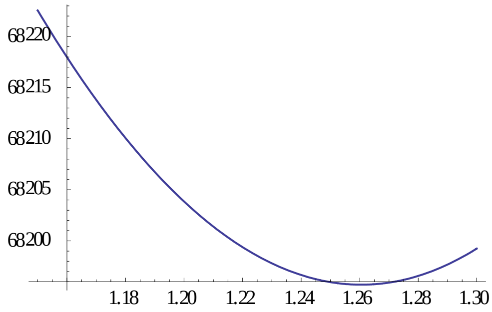
N= 8, F= 1.09959, TC = 7423.6



N = 8, F = 1.09959, TC = 14378.3



N= 10, F = 1.26054, TC = 12239.7



N = 10, F = 1.26054, TC = 24899.2

Figure 5: Showing some best results of example 2

Table 3: Sensitivity analysis of Examples 1 and 2

Parameter	Change in the parameter	F*	% Change in TC(F*) example 1	F*	% Change in TC(F*) example 2
ho	+50	.331844	.0014	1.18231	0.02
	+20	.331844	.0007	1.18231	0.01
	-20	.331844	-.0004	1.18231	-.01
	-50	.331844	-.0004	1.18231	-0.03
hr	+50	.331844	.0006	1.18231	0.00
	+20	.331844	.0006	1.18231	0.00
	-20	.331844	.0006	1.18231	0.00
	-50	.331844	.0007	1.18231	0.00
α	+50	.331709	.0006	1.80330	0.02
	+20	.331709	.0007	1.18152	0.02
	-20	.331898	0.00	1.18310	0.02
	-50	.331989	-.0003	1.18609	-0.05
τ_p	+50	.331844	-0.008	1.18231	-0.05
	+20	.331844	-0.0031	1.18231	-0.02
	-20	.331844	0.004	1.18231	-0.04
Ic	-50	.331844	0.08	1.18231	0.10
	+50	.331004	0.002	1.13050	0.82
	+20	.333199	-0.03	1.6082	0.34
	-20	.332182	-0.01	1.20494	-0.36
Cs	-50	.332690	-0.01	1.24125	-0.93
	+50	.331844	0.00	1.18231	0.00

ISSN 2348-1218 (print)
International Journal of Interdisciplinary Research and Innovations ISSN 2348-1226 (online)
Vol. 7, Issue 2, pp: (587-600), Month: April - June 2019, Available at: www.researchpublish.com

	+20	.331844	0.00	1.18231	0.00
	-20	.331844	0.00	1.18231	0.00
Co	-50	.331844	0.00	1.18231	0.00
	+50	.331844	47.09	1.18231	38.8
	+20	.331844	18.83	1.18231	21.8
	-20	.331844	-18.83	1.18231	-0.01
R	-50	.331844	-47.09	1.18231	-38.8
	+50	.331844	-06.58	1.18231	-33.38
	+20	.331844	21.99	1.18231	-15.87
	-20	.331844	-16.79	1.18231	20.62
δ	-50	.331844	69.34	1.18231	64.61
	+50	.318875	-0.24	1.10073	0.39
	+20	.326418	-0.10	1.14686	0.20
	-20	.337634	0.11	1.22264	0.29
b	-50	.347103	0.29	1.29559	-0.96
	+50	.331844	-31.89	1.01616	-27.12
	+20	.331844	-15.95	1.10873	-13.69
	-20	.331844	23.93	1.26840	20.84
I_e	-50	.331844	95.75	1.42828	84.46
	+50	.331289	-0.44	1.18036	0.006
	+20	.324397	-0.17	1.18153	.0023
	-20	.339305	-0.15	1.18309	-.0028
a	-50	.350373	00.42	1.18426	-.0067
	+50	.331844	42.56	1.18231	42.03
	+20	.331844	17.52	1.18231	17.11
	-20	.331844	-18.23	1.18231	-17.52
λ	-50	.331844	-46.99	1.18231	-44.62
	+50	.331844	0.00	1.18231	-.0003
	+20	.331844	0.000	1.18231	-.0003
	-20	.331844	0.00	1.18231	-.0003
	-50	.331844	0.00	1.18231	-.0003

5. SENSITIVITY ANALYSIS

Using the above example, the decision table F* sensitivity and total cost function TC(F*) was checked in the above table 1 to make changes. Sensitivity analysis is done by changing each parameter from +50. +20, -20 and -50%, is considered one parameter at a time and keeps other parameters unchanged. By doing so it is observed that parameter τ_p, b, a, I_c, C_o and R are extremely sensitive as compared to the other parameters on total cost. Hence, the total cost function TC(F*) is increases or decreases as the parameters h_o, λ, C_s, h_r, δ, α and I_e increases or decreases whereas other parameter shows their reverse effect on total cost function.

6. CONCLUSION

The current study covers the concept of two warehouse and conservation techniques. Analytical results show that there is a unique and improved optimum lot size, which maximizes the expected total profit per time (unit). Finally, after a sensitivity analysis, a numerical example has been presented to clarify the applicability of the proposed model. This model can be used for various types of products like fruit, vegetables, cosmetics etc. The aim of this study is to determine the retailer's optimal replenishment policies that minimizes the total cost. The proposed model has been illustrated by a numerical example and sensitivity analysis.

Further research in this field can be extended in several ways. We can expand the model by considering supply chain or reverse logistics, Weibull distribution etc.

ACKNOWLEDGEMENT

The authors wish to thank the anonymous referees for their helpful comments and suggestions.

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